Exam Introduction to Logic (AI & MA)

Monday 29th January, 2018, 18:30 - 21:30.

☞ Only write your student number at the top of the exam, not your name. Also write your student number at the top of any additional pages. In this way we will be able to grade anonymously.

☞ Use a blue or black pen (so no pencils, red pen or marker).

☞ Leave the first ten lines of the first page blank (this is where the calculation of your grade will be written).

☞ Only hand in your definite answers. You can take any drafts home.

☞ When you hand in your exam, wait until the supervisors have checked whether all information is complete. They will indicate when you can leave.

☞ By writing your student number and tutorial group on all pages, you earn a first 'free' 10 points. With the regular exercises, you can earn 90 points. With the bonus exercise, you can earn 10 extra points. The exam grade is:

> (the number of points you earned with the regular and bonus exercises + the first 'free' 10) divided by 10, with a maximum grade of 10.

☞ The final grade F for the course is computed as

$$F = 0.08 \cdot H_1 + 0.16 \cdot H_2 + 0.16 \cdot M + 0.60 \cdot E.$$

Here H_1 is the grade for homework assignment 1, H_2 is the grade for homework assignment 2, M is the midterm grade, and E is the grade for this final exam.

Good luck!

1: Translating to propositional logic (10 points) Translate the following sentences to *propositional logic*. Atomic sentences are represented by uppercase letters. Provide one translation key for both sentences.

a. They like loud music only if they play jazz.

b. They play jazz unless they miss a drummer and a saxophonist.

2: Translating to first-order logic (10 points) Translate the following sentences to *first-order logic*. Provide one translation key for both sentences. The domain of discourse is the set of all people.

a. Aaron cooks for all his friends who cook for him.

b. If there is someone for whom Barbara cooks, then she cooks for Aaron, who is the only vegetarian among Barbara's friends.

3: Formal proofs (20 points) Give formal proofs of the following inferences. Do not forget the justifications. Remember you can only use the Introduction and Elimination rules and the Reiteration rule.

a.
$$\begin{array}{|l} \neg(A \to B) \\ B \lor C \\ \hline C \end{array}$$

b.
$$\begin{array}{|l} \\ \hline (\neg(A \land B) \lor C) \to (A \to (B \to C)) \end{array}$$

c.
$$\begin{array}{|l} \\ \hline (\exists x P(x) \to Q) \leftrightarrow \forall x(P(x) \to Q) \end{array}$$

with x not occurring free in Q

d.
$$\begin{array}{|l} \exists y \forall x R(x, y) \\ \hline \forall x \exists y R(x, y) \end{array}$$

4: Truth tables (10 points) Answer the following questions using truth tables. Write down the complete truth tables and motivate your answers. Order the rows in the truth tables as follows:

A	B	C	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

a = b	FrontOf(a, b)	BackOf(a, b)	...
T	T	T	...
T	T	F	...
T	F	T	...
T	F	F	...
F	T	T	...
F	T	F	...
F	F	T	...
F	F	F	...

a. Check with a truth table whether the following formula is a tautology.

$$(A \to (\neg B \lor C)) \to ((A \to B) \to \neg(A \land \neg C))$$

b. Check with a truth table whether the conclusion is a logical consequence of the premises for the following argument. Indicate clearly which rows are spurious.

$$\begin{array}{|l} \neg\mathsf{FrontOf}(\mathsf{a}, \mathsf{b}) \\ \neg(a = b) \\ \hline \mathsf{BackOf}(\mathsf{a}, \mathsf{b}) \end{array}$$

5: Normal forms propositional logic (5 points)

Provide a conjunctive normal form (CNF) of the following formula. Show all of the intermediate steps.

$$\neg(A \lor B) \to (\neg A \land \neg(B \lor C))$$

6: Normal forms for first-order logic and Horn sentences (10 points)

a. Provide a Prenex normal form of the following sentence

$$\neg(\exists x P(x) \lor \forall x \forall y \exists z Q(x, y, z))$$

Show all intermediate steps.

b. Provide a Skolem normal form of the following sentence

$$\forall x \exists y \forall z \forall w \exists v (R(x, y) \vee (Q(y, z, w) \wedge \neg P(v))$$

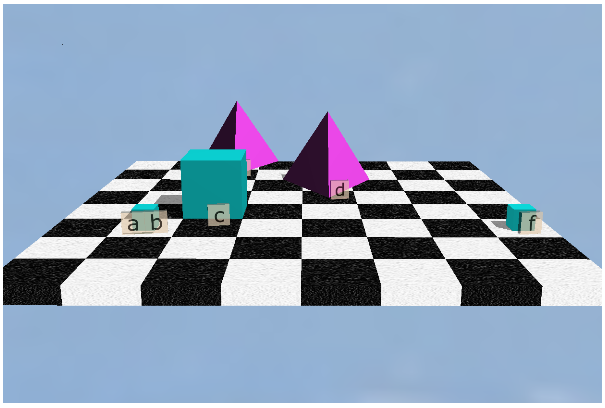
Show all intermediate steps.

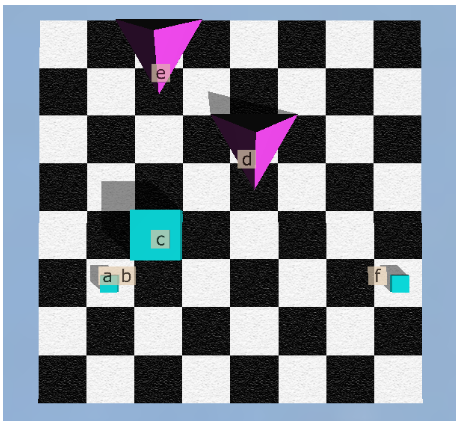
c. Check the satisfiability of the following Horn sentence.

$$\neg A \wedge B \wedge (C \vee \neg B) \wedge (\neg C \vee \neg D \vee E) \wedge (\neg B \vee \neg E)$$

Use the Horn algorithm and indicate the order in which you assign truth values to the atomic sentences. If you prefer the conditional form, you may also use the satisfiability algorithm for conditional Horn sentences.

7: Tarski's World (10 points)





In the world displayed above, a, b and f are small, e and d are large and c is medium sized.

a. In the world displayed above, all and only the tetrahedrons are large. Express this with one sentence in the language of Tarski's World. The sentence should be true in all worlds in which all tetrahedrons are large and no other object is large, and false in all worlds in which either some tetrahedron is not large or some other object is large.

b. Indicate of the sentences below, whether they are true or false in the world displayed above. You do not need to explain your answers.

(i) $\mathsf{SameShape(a, b)} \to (\neg\mathsf{Larger(a, b)} \wedge \neg\mathsf{Larger(b, a)})$

(ii) $\forall x(\mathsf{Dodec}(x) \to \mathsf{Small}(x))$

(iii) $\exists x \exists y \exists z(\mathsf{SameShape}(x, y) \wedge \mathsf{SameShape}(y, z) \wedge \mathsf{Between}(x, y, z))$

(iv) $\forall x \neg \exists y((x \neq y \wedge \mathsf{SameShape}(x, y)) \to (\mathsf{FrontOf}(y, x) \to \mathsf{Larger}(y, x)))$

c. Explain how the formula below can be made **false** by changing the position of one object in the world displayed above. Explain which object to move and to which position by saying how many cells to the right or left, and how many cells up or down the object should be moved.

$$\forall \mathsf{x} \forall \mathsf{y}((\mathsf{FrontOf(y, x)} \wedge \mathsf{SameColumn(x, y)}) \to \exists \mathsf{z}(\mathsf{SameShape(z, y)} \wedge \mathsf{FrontOf(z, y)} \wedge \neg\mathsf{Larger(z, x)}))$$

8: Translating function symbols (5 points) Translate the following sentences using the translation key provided. The domain of discourse is the set of all Renaissance painters.

r: Raphael
m: Michelangelo

firstmaster(x): x's first master
chiefrival(x): x's chief rival

Morefamous(x, y): x is more famous than y

a. Raphael's first master is more famous than Michelangelo's first master but there is someone's first master who is more famous than both Michelangelo's and Raphael's first masters.

b. Michelangelo is Raphael's chief rival if and only if Raphael is Michelangelo's chief rival.

9: Semantics (10 points)

Let a model $\mathfrak{M}$ with domain $\mathfrak{M}(\forall) = \{\text{Rock, Scissors, Paper}\}$ be given such that

- $\mathfrak{M}(a) = \text{Rock}$
- $\mathfrak{M}(\mathsf{Frequent}) = \{ \text{ Rock } \}$
- $\mathfrak{M}(\mathsf{Infrequent}) = \{ \text{ Scissors } \}$
- $\mathfrak{M}(\mathsf{Beats}) = \{\langle\text{Rock, Scissors}\rangle, \langle\text{Scissors, Paper}\rangle, \langle\text{Paper, Rock}\rangle\}$

Let h be an assignment such that:

- $h(x) = \text{Rock}$,
- $h(y) = \text{Scissors}$,
- $h(z) = \text{Paper}$.

Evaluate the following statements. Follow the truth definition step by step.

a. $\mathfrak{M} \models \mathsf{Frequent}(x) \to (\mathsf{Infrequent}(y) \to \mathsf{Beats}(x, y))[h]$

b. $\mathfrak{M} \models \exists x(\neg\mathsf{Infrequent}(x) \wedge \mathsf{Beats}(x, a))[h]$

c. $\mathfrak{M} \models \exists x\forall y\mathsf{Beats}(x, y)[h]$

10: Bonus question (10 points)

Give a formal proof of the following inference. Don't forget to provide justifications. You can only use the Introduction and Elimination rules and the Reiteration rule.

$$\begin{array}{|l} \forall x\exists yR(x, y) \to \exists y\forall xR(x, y) \\ \hline \exists x(\forall y\neg R(x, y) \vee \forall yR(y, x)) \end{array}$$